

Exercice 2.10

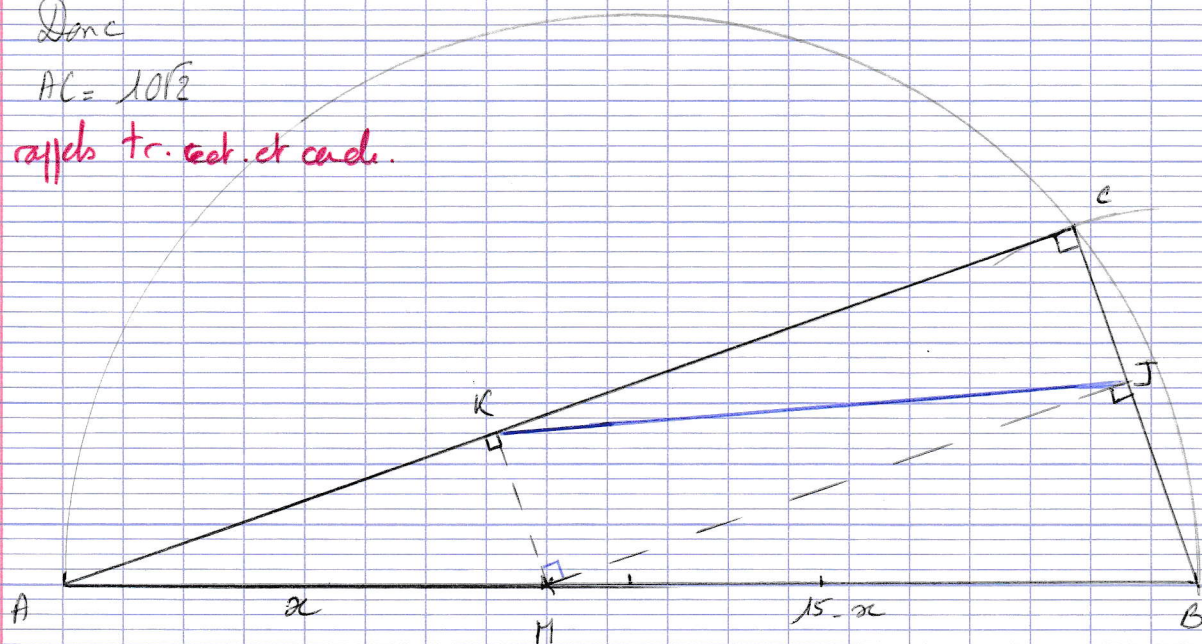
Dans $\triangle ABC$ rectangle en C :

$$AC^2 = 15^2 - 5^2 = 225 - 25 = 200$$

Donc

$$AC = 10\sqrt{2}$$

⊗ rappels tr. rect. et cercle.



Les triangles AKN et NJB sont semblables, on a donc

$$\frac{NJ}{AK} = \frac{JB}{KN} = \frac{NB}{AN}$$

① Calcul de KN .

$$\frac{JB}{KN} = \frac{NB}{AN}$$

$$\frac{5 - CJ}{KN} = \frac{15 - x}{x}$$

$$\frac{5 - KN}{KN} = \frac{15 - x}{x}$$

⊗ rapd pds en croix

$$(5 - KN)x = (15 - x)KN$$

$$5x - KNx = 15KN - KNx$$

$$KN = \frac{5x}{15}$$

$$\parallel KN = \frac{x}{3}$$

② Calcul de NJ .

$$\frac{NJ}{AK} = \frac{NB}{AN}$$

$$\frac{NJ}{10\sqrt{2} - KC} = \frac{15 - x}{x}$$

$$\frac{NJ}{10\sqrt{2} - NJ} = \frac{15 - x}{x}$$

$$NJx = (15 - x)(10\sqrt{2} - NJ)$$

$$NJx = 150\sqrt{2} - 15NJ - 10\sqrt{2}x + NJx$$

$$15NJ = 150\sqrt{2} - 10\sqrt{2}x$$

$$NJ = \frac{150\sqrt{2} - 10\sqrt{2}x}{15}$$

$$\parallel NJ = \frac{30\sqrt{2} - 2\sqrt{2}x}{3}$$

③ Calcul de KJ^2

Dans KNT rectangle en N, on a:

$$KJ^2 = KN^2 + NJ^2$$

$$= \left(\frac{x}{3}\right)^2 + \left(\frac{30\sqrt{2} - 2\sqrt{2}x}{3}\right)^2$$

$$= \frac{1}{9} \left(x^2 + 900 \times 2 - 2 \times 30\sqrt{2} \times 2\sqrt{2}x + (2\sqrt{2}x)^2 \right)$$

$$= \frac{1}{9} (x^2 + 1800 - 240x + 8x^2)$$

$$= \frac{1}{9} (9x^2 - 240x + 1800)$$

$$= x^2 - \frac{80}{3}x + 200$$

On trace la courbe, le minimum est atteint pour

$$x_0 = \frac{80}{6} = \frac{40}{3} \approx 13,33 \text{ cm.}$$

